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A STUDY ON THE EIGENMODES OF THE SQUARE AND CIRCULAR COAXIAL WAVEGUIDES BY FEM

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KEYWORDS: coaxial waveguide, eigen-mode, Krylov-Schur iteration, TE and TM mode, dielectrics, Schur form.

ABSTRACT
FEM has been used to construct an eigen-equation for coaxial waveguides. Their cross sections have been square and circular shapes made with different dielectric constants. Krylov-Schur iteration method has been carried out to obtain the eigen-modes for these waveguides. The eigen-modes for each waveguides have been divided into two classes of TE and TM modes. These eigen-modes have been schematically represented with the spectra of transverse vector fields and their corresponding vector potential values. The spectra have shown the distribution of fields surrounding the interface between two dielectrics. Especially, the spectra have revealed the complicated distribution of field at the inner region of the dielectrics. As increased the distance from the center, the intensity of electromagnetic wave is decreased and arranged in the structured form. From these results, it could be identified that the eigen-modes seemed reflecting on the dielectric properties of coaxial waveguides.

INTRODUCTION
From the beginning of optical communication, there have been a lot of efforts for designing an ideal waveguide. The waveguide must be able to propagate the stable eigen-mode through the long distance without any distortion. The waveguide has not to permit any power leakage over its boundary. By Snell's law, the propagating wave can be confined in the specific region of the greater dielectric constant than the area of the smaller value [1]. The distribution of dielectrics can be established in the various way. Among the various, the one would be the coaxial distribution having the greater dielectric constant in the core region than the outer area. The coaxial waveguide has matched with these requirement and attracted many attentions from applied fields [2]. In engineering application, the most popular design may be the rectangular or circular forms with the greater dielectric constant in the core region of the waveguide [3].

The eigen-property of the waveguide is a matter that must be considered firstly than others. Physically, this property may be revealed by the eigen-mode and eigen-value which is the most important thing characterizing the waveguide. These are directly related to the geometrical structure of waveguide and the distribution of dielectrics in its space. The eigen-properties for above mentioned waveguides have been investigated variously with theoretical and experimental methods. So far, for a long time, a lot of information about the eigen-mode have been accumulated and applied in the actual communication system. But in some case, the theoretical method could not explain the eigen-properties including the influence of the interface between the different dielectrics. In particular, the high order eigen-modes obtained through experiments have been shown a complicated form that cannot be theoretically analysis. This phenomenon has been appeared even more seriously from the coaxial waveguide of complicated structure. Generally, the numerical analysis has been applied to overcome this problem. Among others, FEM(Finite element method) has been well known as one of the most reliable method to resolve such a problem. Previously, we have studied on the eigen-modes of square waveguide made with two different dielectrics. The distribution of dielectrics was simply a step-like along the transvers direction of the waveguide. In there, the eign-pairs have been obtained and compared them with the theoretically calculated results [4].

As an extension of this study, the similar calculation have applied to waveguides of the some more complicated structure. Waveguides were square and circular coaxial forms with the greater dielectric constant in the core region. In the course of FEM analysis, the cross section was divided into triangular mesh. The eigen-matrix equation was constructed using tangential edges and nodes of triangular element meshes. Tangential edges and nodes were used as calculating variables for transverse vector fields and their vector potential component respectively. The eigen-modes were obtained by using the Krylov-Schur iteration method [5]. TE(Transverse Electric) and TM(Transverse magnetic) modes could be calculated by giving different boundary conditions on the surface of the waveguide. Each columns of the similarity transforming matrix have contained the eigen-modes of transverse vector field and its vector potential component simultaneously. As a result, among the eigen-modes,

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several prominent TE and TM vector fields accompanying with their vector potential component were revealed by schematic presentation.
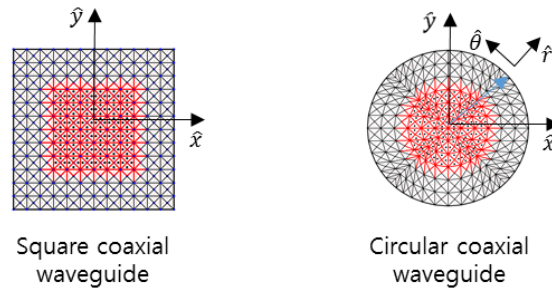
FINITE ELEMENT FORMULATION

Fig. 1 The Cross sectional form of waveguides.

The eigen-modes of the waveguides depends on the geometric structure and the distribution of the dielectric constant. In this study, the waveguides are assumed to be square and circular coaxial forms as fig.1. The dielectric constant is the greater in the core region than the outer area without differentiate the geometric structure. Process for calculating the eigen-modes is the same as describing in reference [4]. The vector Helmholtz equation would be used in determining the wave property of waveguides. It is described as following equation [6] [7]

$$\vec{\nabla} \times \left(p\nabla \times \vec{F} \right) - k_o^2 q\vec{F} = 0 \qquad (1)$$

Where k_o is the wave number and, for $\vec{F} = \vec{E}$(electric field strength), $p = 1/\mu_r$(μ_r: relative permeabilityμ/μ_o), $q = \varepsilon_r$(ε_r: relative permittivity $\varepsilon/\varepsilon_o$) and, for $\vec{F} = \vec{H}$ (magnetic field strength), $p = 1/\varepsilon_r$, $q = \mu_r$. For convenience of calculation, the common notations $\vec{F}$, p and q would be used to relate these values without differentiating electromagnetic field modes. The eigen-equation is constructed from FEM based on the triangular elementary mesh. The shape functions for the triangular element mesh are calculated as following

$$\begin{Bmatrix} N_1 \\ N_2 \\ N_3 \end{Bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{bmatrix}^{-1} \begin{Bmatrix} 1 \\ x \\ y \end{Bmatrix} = \frac{1}{2A}\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}\begin{Bmatrix} 1 \\ x \\ y \end{Bmatrix} \qquad (2)$$

where $a_i = x_j y_k - x_k y_j, b_i = y_j - y_k, c_i = x_k - x_j$ (i, j and k are cyclical ordering) and A is the area of the triangular element mesh. Relating with these shape functions, the constant tangential edge vectors for the element mesh are given by

$$\vec{W_m} = L_m \left(N_i \vec{\nabla}_t N_j - N_j \vec{\nabla}_t N_i \right) \qquad (3)$$

where $\vec{\nabla}_t = \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y}$, $L_m$is the length connecting nodes i and j. It is more convenient to express these vectors by using the shape function coordinates.

$$\vec{W_m} = \frac{L_m}{4A^2}[(A_m + B_m y)\hat{x} + (C_m + B_m x)\hat{y}] \qquad (4)$$

where $A_m = a_i b_j - a_j b_i$
$B_m = c_i b_j - c_j b_i$
$C_m = a_i c_j - a_j b_i$
$D_m = b_i c_j - b_j c_i = -B_m$

Global Journal of Engineering Science and Research Management

With these tangential edge vectors and shape functions, the transverse vector fields and their vector potential component of each element mesh can be written as

$$\begin{Bmatrix} F_x \\ F_y \\ F_z \end{Bmatrix} = \begin{bmatrix} \{W_x\}^T & 0 \\ \{W_y\}^T & 0 \\ 0 & j\{N\}^T \end{bmatrix} \begin{bmatrix} \{F_t^e\} \\ \{F_z^e\} \end{bmatrix} \tag{5}$$

The Galerkin method of weighted residual has been used to construct the eigen-equation. The envelope function concept for the element vector

$$\{F\} = \begin{Bmatrix} \vec{F_t} \\ F_z' \end{Bmatrix} \exp(-j\beta)$$

is used where β is a propagation constant. The eigen-equation was obtained from the (0,1) Pade approximation of propagation scheme at the beginning position[8]

$$[A]\{F\} = -\frac{1}{2k_o n\beta}[B]\{F\} \tag{7}$$

where $[A] = \begin{bmatrix} [G] & [E] \\ [F] & [D] - k_o^2[I] \end{bmatrix}$

$[B] = \begin{bmatrix} k_o^2 n_o^2[G] - k_o^2[H] + [C] & k_o^2 n_o^2[E] \\ k_o^2 n_o^2[F] & k_o^2 n_o^2([D] - k_o^2[I]) \end{bmatrix}$

and for the calculation stability $F_z' = j\frac{\partial}{\partial z}F_z$ was adopted. The matrices components is calculated as following

$[C] = \iint_A p(\vec{\nabla}_t \times \vec{W_m}) \cdot (\vec{\nabla}_t \times \vec{W_n})ds = p\frac{L_m L_n}{4A^3}D_m D_n$

$[D] = \iint_A p(\vec{\nabla}N_i) \cdot (\vec{\nabla}N_j)ds = p\frac{b_i b_j + c_i c_j}{4A}$

$[E] = \iint_A p\vec{W_m} \cdot \vec{\nabla}N_j ds = p\frac{L_m}{8A^2}[b_j(A_m + B_m\bar{y}_{tri}) + c_j(C_m + D_m\bar{x}_{tri})$

$[F] = \iint_A p\vec{\nabla}N_i \cdot \vec{W_n}ds = p\frac{L_n}{8A^2}[b_i(A_n + B_n\bar{y}_{tri}) + c_i(C_n + D_n\bar{x}_{tri})$

$[G] = \iint_A p\vec{W_m} \cdot \vec{W_n}ds = p\frac{L_m L_n}{16A^3}\sum_{a=1}^{a=5} I_a$

where $I_1 = (A_m A_n + C_m C_n)$
$I_2 = (C_m D_n + D_m C_n)\bar{x}_{tri}$
$I_3 = (A_m B_n + B_m A_n)\bar{y}_{tri}$
$I_4 = \frac{B_m B_n}{12}(\sum_{i=1}^{i=3} y_i^2 - 9\bar{y}_{tri}^2)$
$I_5 = \frac{D_m D_n}{12}(\sum_{i=1}^{i=3} x_i^2 - 9\bar{x}_{tri}^2)$

$[H] = \iint_A q\vec{W_m} \cdot \vec{W_n}ds = q\frac{L_m L_n}{16A^3}\sum_{a=1}^{a=5} I_a$

$[I] = \iint_A N_i N_j ds = q\frac{A}{12}\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$

In these representations, subscripts for N and $\vec{W}$ indicate the node and edge numbers respectively. These element matrices have been assembled over all triangular elements to obtain a global eigen-matrix equation.

As mentioned in the previous study, it has been well known that the Krylov-Schur iteration method is the most reliable technique for finding the prominent eigen-modes [9]. The method would be more efficiently implemented in finding specific eigen-pairs by performing the shift-invert strategy as following [10]

$$\lambda_o\{F\} = \frac{[B]}{[A] - \sigma[B]}\{F\} = [M]\{F\} \qquad (8)$$

where $\lambda_o = \frac{1}{-\frac{1}{2nk_0\beta} - \sigma}$. The sparsity and symmetry of the eigen-equation would be lost, but by this strategy the convergent rate is more promoted at the specific value σ. Subsequently, the Krylov-Schur iteration method is performed on this square matrix [M].

RESULTS AND DISCUSSION

TE mode   TM mode
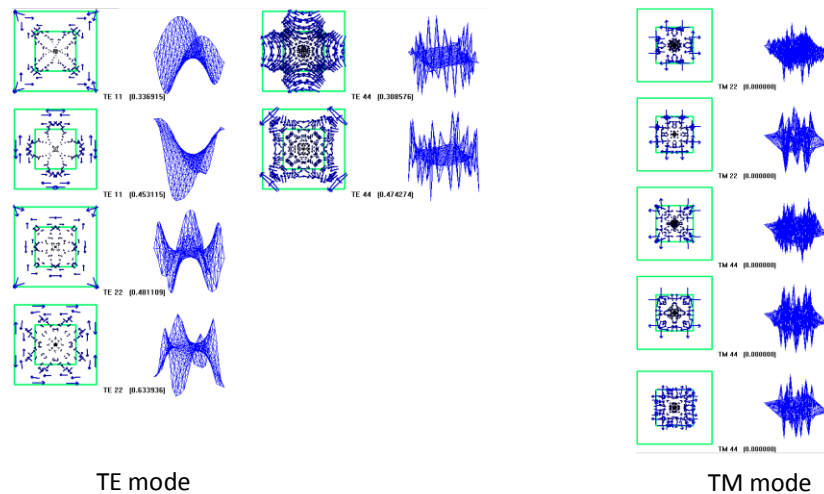
Fig. 2 Eigen modes of the square coaxial waveguide.

In this study, FEM was carried out to investigate the eigen-properties of coaxial waveguides. Eigen-modes are depend on the geometric structure of the waveguide. The number of mesh determine the accuracy of the calculation. However, the shape of mesh does not affect the accuracy of the calculation process. Generally, triangular mesh for square waveguides is easier to configure than the circular waveguide. The triangular mesh for a square waveguide were configured firstly and then transformed it to the shape of modified triangular mesh for the circular waveguide as like seen in fig.1. Each waveguides were consisted of two dielectrics having different dielectric constant. The relative dielectric constant of the core and outer regions were assumed to be 1.5 and 1.0 respectively. As in the previous study, the lateral surface of the waveguides were assumed to be perfect conductor. The reason for it was that this assumption is convenient to obtain TM modes by ignoring the variables on the surface. From these assumption, the matrix form of the eigen-equation was constructed as like eq.8. The eigen-pairs were calculated by performing the Krylov-Schur iteration on the square matrix $[M]$ of eq. 8. As a result, spectra are revealed schematically in fig.2, 3.
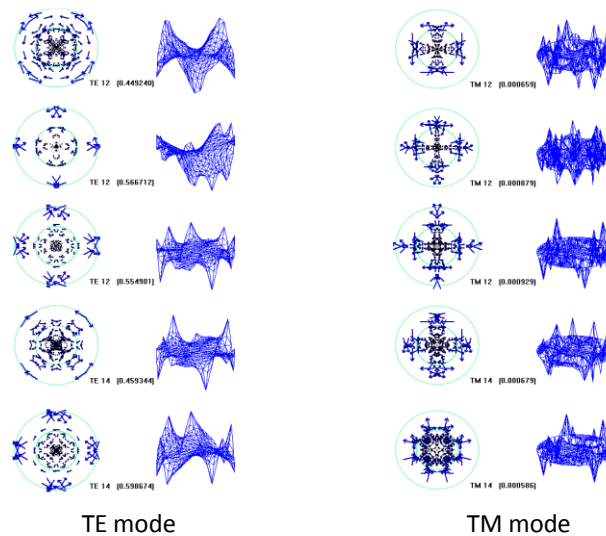
TE mode   TM mode

Fig. 3 Eigen modes of the circular coaxial waveguide.

Fig. 2 and fig. 3 are the spectra for the square and circular coaxial waveguides respectively. The eigen-modes were the column vectors of the similar transforming matrix which convert the square matrix $[M]$ to a Shure form. The eigen-values were calculated by converting each diagonal component of the Schur matrix into value $-\frac{1}{2nk_o\beta} = \frac{1}{\lambda_o} + \sigma$ reversing the shift-invert strategy. The propagation constants calculated from this relation were written in the blanket under each spectrum. The reflectivity n was assumed to be an average value of the waveguide. The wave number was set to be $k_o = 1$ for the sake of convenience. The eigen-modes have been represented with two components. The one is for the transverse electromagnetic vector field. The other is the vector potential component for the transverse electromagnetic vector field. The former was resulted from the tangential edge vectors and the latter was obtained from the nodes of the triangular element. The left columns of (a), (b) for fig. 2, 3 respectively, represents transverse modes which have been obtained by applying eq. 5 to corresponding tangential edge variables of the triangular elements. The right columns of (a), (b) for fig. 2, 3 respectively, represent components of their corresponding vector potential resulted by applying eq. 5 to their node variables. Spectra (a), (b) in fig. 2, 3 represent TE and TM modes of waveguides respectively. Mode types were easily determined by counting the peak distribution of the vector potential component for low order eigen-modes. As the mode order increased, the mode type could not be easily determined because peaks of the vector potential components are mixed complicated. The electromagnetic wave is concentrated in the core region where dielectric constant is larger than the outer area. As increased the distance from the center, the intensity of electromagnetic wave is decreased and arranged in the structured form. As can be seen in figures, these spectra show reliably other aspects comparing to spectra of the previous study. But, this feature can be related with the action of the interface between the dielectrics. It has been identified from the previous study that the interface of dielectrics dominantly affect the characteristics of the eigen-modes of waveguides. In there, it has been shown that the electromagnetic wave was stacked tightly around the interface. This phenomenon would take place in the same manner for the coaxial waveguide. The core region is surrounded by the interface which would be complicating the eigen-properties in there. Such a phenomenon is remarkable in TM mode than TE mode of both waveguides. The reason for it could be identified from the distribution of dielectric constant and the boundary condition of TM mode. By the Snell's law, the interface between the dielectrics reflect the incident beam with the angle larger than critical angle. This action concentrate electromagnetic wave into the region of the greater dielectric constant. In the application, the waveguide is designed to be coaxial structure to implement this purpose. This characteristic can also eliminate leakage of energy of the electromagnetic wave in the process of transmitting a signal in the far away. On the other hand, the lateral surface of waveguides was assumed to be a perfect conductor. This assumption made it possible to ignore the variables for TM mode at the waveguide surface. Electromagnetic waves are excluded from the surface and drive fields to exists densely only inside the waveguide. This assumption would be another reason for concentrating electromagnetic wave into the inner region. So, it has been understood that these reason is reflected in the spectra of TM modes.

Global Journal of Engineering Science and Research Management

CONCLUSION

FEM has been carried out to investigate the eigen-properties of the square and circular waveguides. Krylov-Schur iteration method was applied to calculate the eigen-modes of TE, TM and their vector potential components. The schematic representation for these spectra have shown that the electromagnetic wave is concentrated in the core region of the greater dielectric constant. From the spectra, it has been identified that eigen-modes could be easily determined by counting the vector peaks of the potential components. By comparing to the previous study, it is identified that the eigen-modes are dominantly influenced by the interface.

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